Mathematics : The core course for A-level : Bostock & Chandler.

Chapter 7 : Trigonometric Identities

Exercise 7b

(1) $\sin(A+B) = \sin A \cos B + \sin B \cos A$

With B Replaced by $-B$ we have

$$\sin(A+(-B)) = \sin A \cos(-B) + \sin(-B)\cos A$$

But, in general, $\cos(-\theta) = \cos\theta$
and $\sin(-\theta) = -\sin\theta$

So

$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$

(2) $\sin(A-B) = \sin A \cos B - \sin B \cos A$

Replace A by $\frac{\pi}{2} - A$ to get

$$\sin\left(\frac{\pi}{2} - A - B\right) = \sin\left(\frac{\pi}{2} - A\right)\cos B - \sin B \cos\left(\frac{\pi}{2} - A\right)$$

let $D = A+B$:

$$\sin\left(\frac{\pi}{2} - D\right) = \cos A \cos B - \sin B . \sin A$$

But $\sin\left(\frac{\pi}{2} - D\right) = \cos D = \cos(A+B)$ So

$$\cos(A+B) = \cos A \cos B - \sin A \sin B.$$

(3) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

Let B be replaced by $-B$. Then

$$\cos(A + (-B)) = \cos A \cos(-B) - \sin A \sin(-B)$$

But in general we have $\cos(-\theta) = \cos\theta$
& $\sin(-\theta) = -\sin\theta$

So

$$\cos(A - B) = \cos A \cos B + \sin A \sin B.$$

(4) $$\frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B - \sin A \sin B}$$

Divide RHS by $\cos B$:

$$\tan(A+B) = \frac{\sin A + \tan B \cos A}{\cos A - \sin A \tan B}$$

Now divide numerator and denominator of RHS by $\cos A$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

(5) Let B be Replaced by $-B$. Then

$$\tan\left(A+(-B)\right) = \frac{\tan A + \tan(-B)}{1 - \tan A \tan(-B)}$$

But $\tan(-B) = \dfrac{\sin(-B)}{\cos(-B)} = \dfrac{-\sin B}{\cos B} = -\tan B$

Hence

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$